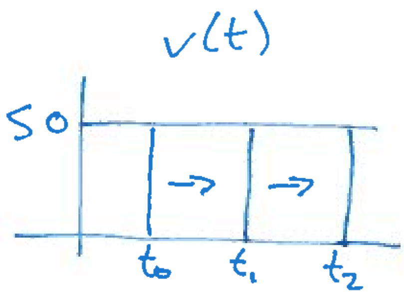
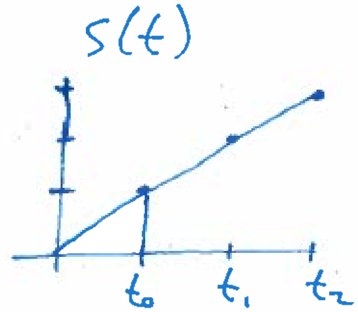
Chapter 5: Integrals

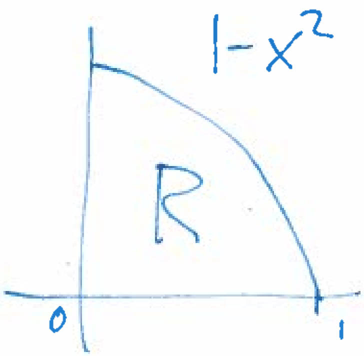
5.1: Area and Estimating with Finite Sums

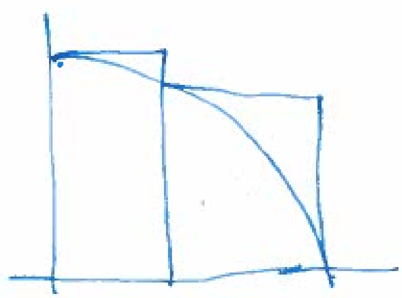
Car driving 50 mph. $v(t) = 50$. So $s(t) = 50t$.

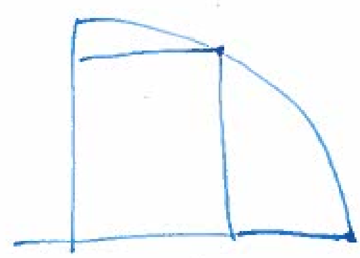




Estimating Area: Riemann Sums







5 types: Left, Right, Lower, Upper, Midpoint.

More rectangles = Better Approximation

5.2: Sigma Notation and Limits of Finite Sums

Also known as Summation Notation.

Capital Sigma $\rightarrow$ $\displaystyle\sum_{K=1}^{n} a_K = a_1 + a_2 + \cdots + a_{n-1} + a_n$

$n \leftarrow$ end point.

$K=1 \leftarrow$ starting point

$K \leftarrow$ Index of summation

Ex: (1) $\displaystyle\sum_{K=1}^{11} K^2 = 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2+11^2$

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(2) $\displaystyle\sum_{K=4}^{11} K^2 = 4^2+5^2+6^2+7^2+8^2+9^2+10^2+11^2.$

(3) $\displaystyle\sum_{K=1}^{3} (-1)^K \cdot K = (-1)^1 \cdot 1 + (-1)^2 \cdot 2 + (-1)^3 \cdot 3 = -1+2-3 = -2$

Rules: (1) $\displaystyle\sum_{K=1}^{n} (a_K \pm b_K) = \sum_{K=1}^{n} a_K \pm \sum_{K=1}^{n} b_K$

(2) $\displaystyle\sum_{K=1}^{n} c \cdot a_K = c \cdot \sum_{K=1}^{n} a_K$

(3) $\displaystyle\sum_{K=1}^{n} c = n \cdot K.$

Ex(4): Show that $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

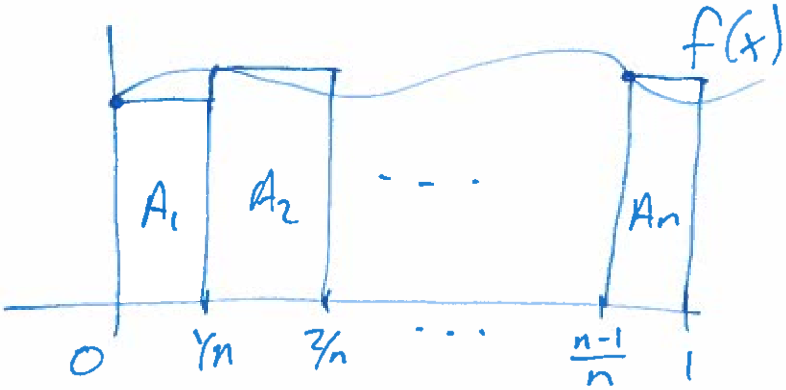
$$2\sum_{k=1}^{n} k =$$

$$\begin{array}{ccccccccc} & 1 & +2 & +3+ & \cdots & +n-1 & +n \\ + & n & +n-1 & +n-2+ & \cdots & +2 & +1 \\ \hline \end{array}$$

$$\underbrace{n+1 + n+1 + \cdots + n+1 + n+1}_{n \text{ times}}$$

$= n(n+1)$. Thus $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$.

Applying to Riemann Sums (w/ limits)



Estimate area under curve, from left, with n subintervals.

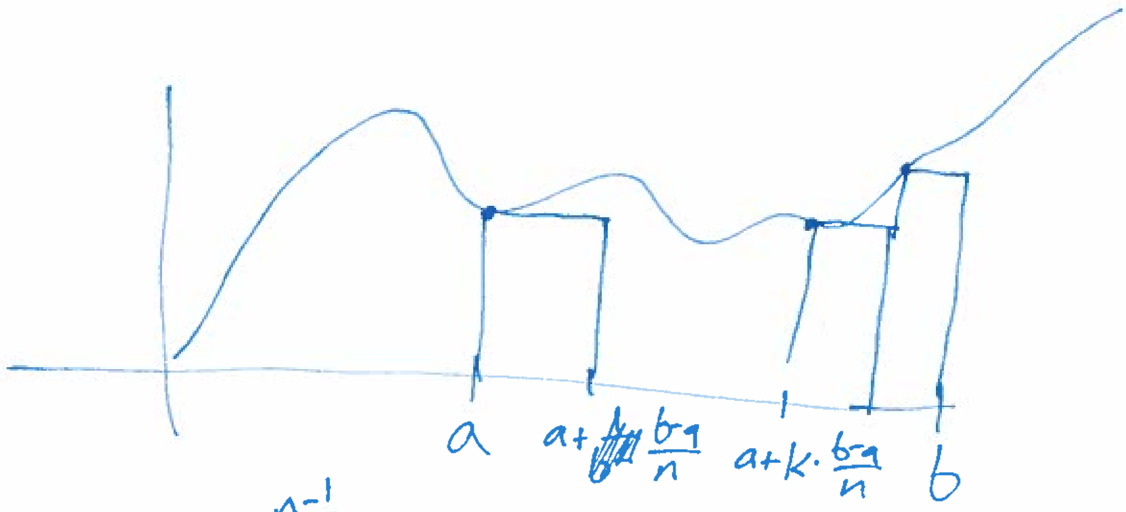
$A_1 = f(0)\cdot\frac{1}{n}$, $A_2 = f(\frac{1}{n})\cdot\frac{1}{n}$, $\ldots$, $A_n = f(\frac{n-1}{n})\cdot\frac{1}{n}$.

So Total Area $= \sum_{k=1}^{n} A_k = \sum_{k=1}^{n} f\left(\frac{k-1}{n}\right)\cdot\frac{1}{n}$.

To get rid of ~~area~~ error, we take limit $n \to \infty$.

So Actual Area $= \lim_{n\to\infty} \sum_{k=1}^{n} f\left(\frac{k-1}{n}\right)\cdot\frac{1}{n}$.

Estimate area of general interval $[a,b]$ w/ n subintervals



So $\text{Area} = \sum_{k=0}^{n-1} f\left(a+k\cdot\frac{b-a}{n}\right)\cdot\frac{b-a}{n} = \sum_{k=1}^{n} f\left(a+(k-1)\frac{b-a}{n}\right)\cdot\frac{b-a}{n}.$